

AN APPLICATION OF THE INDEPENDENT COMPONENT ANALYSIS METHODOLOGY TO GAMMA RAY ASTROPHYSICAL IMAGING

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Abstract

Independent Component Analysis (ICA) [1] is a statistical method often used to decompose a complex dataset in its independent sub-parts. It is a powerful technique to solve a typical Blind Source Separation problem. A fast calculation of the gamma ray sky observed by GLAST, assuming the expected instrumental response, has been implemented. The simulated images were used to test the capability of the ICA method in identifying the sources.

1 Introduction

Maps produced in large area surveys contain a linear mixture of signals from several astrophysical and cosmological sources convoluted with the spatial and spectral response of the detector. GLAST, thanks to its sensitivity (about 50 times better than EGRET at $E > 100$ MeV) and its wide field of view (more than $2.5sr$), will reveal a large number of sources during its lifetime, giving scientists a superior data sample to study many interesting phenomena.

A common problem encountered is to detect and analyze these sources. Many classical tools (i.e. likelihood) need assumptions on a theoretical model in order to fit the data and to extract physical parameters.

To optimize the computation time and to minimize the data modelling assumptions, an alternative technique based on Independent Component Analysis has been investigated. It could be a valid tool to identify signals of different origin in the sky maps. A previous application of ICA in the astrophysical domain is due to Baccigalupi *et al.* [2], to separate the contribution of Cosmic Microwave Background in Planck simulated images.

In section 2 we will outline the ICA methodology to extract an independent component from a set of linearly mixed sources, while in section 3 a preliminary study of the ICA performance on gamma ray source spatial information will be described.

2 ICA for astrophysical images

The method requires that the independent components are statistical independence and at most one of them is gaussian. For astrophysical images the previous conditions are theoretically satisfied and also the linear model holds exactly.

Let us denote with $\mathbf{s} \equiv (\mathbf{s}_1, \dots, \mathbf{s}_m)^T$ the vector of the M signal sources, where each \mathbf{s}_i is the individual image of the i -th source (with the total T pixels stacked row by row into a T -vector), and with $\mathbf{x} \equiv (\mathbf{x}_1, \dots, \mathbf{x}_n)^T$ the vector of the observed signal in N different energy bands, where each \mathbf{x}_i is a T -vector as above. The analytical form for the ICA model is $x = As$, where \mathbf{A} is a mixing matrix to be determined. Using the central limit theorem, ICA is able to solve the problem. Let us define a matrix \mathbf{W} such that the transformed \mathbf{y} vectors are as independent as possible

$$\hat{s} = y = Wx \quad (1)$$

We can derive the independent component \mathbf{s} by minimizing the dependence of the \mathbf{y} vectors, that means, for the central limit theorem, to maximize the non gaussianity of the $\mathbf{W}\mathbf{x}$ vector. The FastICA [4] algorithm allows to find a matrix \mathbf{W} as the best estimator of the inverse matrix \mathbf{A}^{-1} , minimizing the negentropy function, that gives a quantitative measure of the gaussianity, for the \mathbf{y} variable. The columns of the \mathbf{W} matrix are updated by the iteration:

$$w' = E\{xg(w^T x)\} - E\{g'(w^T x)\}w \quad (2)$$

and then the matrix \mathbf{W} is orthonormalized. The function g in equation (2) is an odd nonlinear function and g' its derivative, we assume $g(u) = \tanh(u)$. Once the algorithm has converged, the estimation of the single components can be obtained using equation (1).

3 FastICA and simulated GLAST maps

In order to test the capability of the ICA method in identifying the dominant contributions in an astrophysical map, images of a chosen region of the sky, as observed by the GLAST telescope, have been simulated using the GLAST Light Simulator program described in [3]. Maps of 41×41 or 21×21 (T -dimension=1681 or 441) pixels ($1px = 0.5^\circ$) in seven different energy ranges (N -dimension=7), between 10 MeV and 1 GeV, have been generated. The independent simulated components included in the maps are: the diffuse background (galactic and extragalactic), the faint sources generated randomly and the sources from the Third Egret Catalogue.

The performance of the algorithm for different signal (sources) to noise (diffuse background) conditions have been tested.

First images in a sky region around blazar 3C279 with a long exposure period (one precession period ~ 54 days) have been simulated; in this region there is a large contribution from the sources (fig.1) because we are far from the galactic plane, where the background is dominant. The method works correctly and the position of the sources in the region are correctly reconstructed (fig.2). Reducing

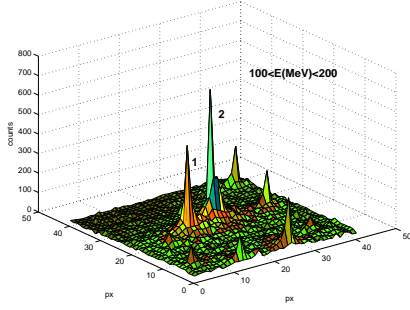


Figure 1: One of the input images around 3C279 and ~ 54 day of exposure.

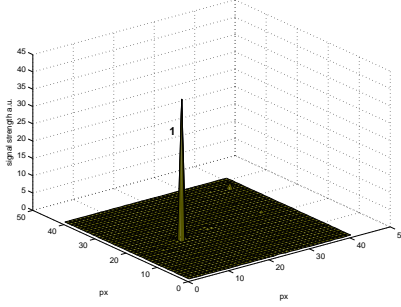


Figure 2: One of the output images: the source 1 has been identified

the windows to 21×21 pixels, results on information also about the faint sources, even if their contribution is very small because of the presence of the brighter ones. Subtracting the contribution of latter would allow this technique to identify sources in the shadow. Looking at the same region, after reducing the number of photons from the sources by a factor 150, in order to have comparable contributions from signal and noise, the method is always able to extract the dominant components (fig.3).

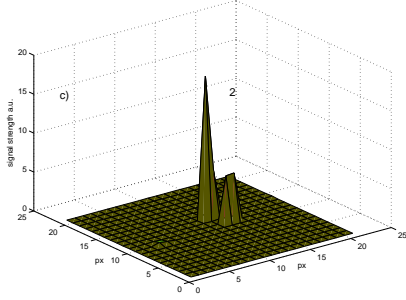


Figure 3: Number of photons from the sources reduced by a factor 150. One of the output images: source 2 has been identified

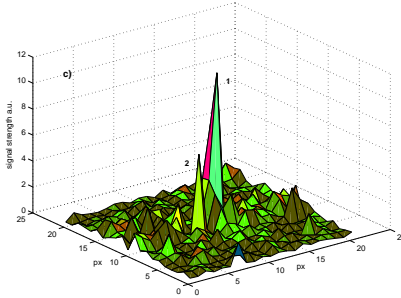


Figure 4: Region around the Galactic Center. One of the output images: sources 1 and 2 are identified

In a second step, the observation windows has been moved towards the galactic center and an exposure of about 10 days has been simulated. In this region the contribution from the diffuse background is considerable larger, nevertheless the method is able to correctly reconstruct the position of the sources (fig.4).

In the previous cases the method identifies the sources but sometimes some of them are reconstructed together as a single component, especially if they have

peaks with very low intensity. The following test case has been studied to test the capability of ICA to separate sources in presence of a superposition. In the input images the distance between the peaks of the sources has been reduced to about 1 pixel. As shown in fig.5, each source is distinguished in a component, despite of the small distance between them in the input maps.

In this first application of the ICA method, information about the signal intensity have not been carried out, but only indications about the position of the sources have been studied.

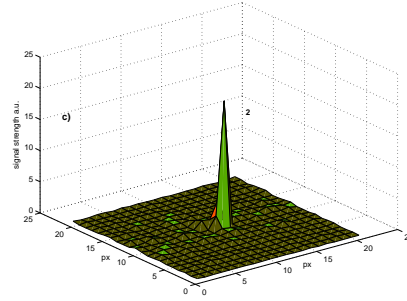


Figure 5: Superposition of the sources. One of the output images: source 2 has been identified.

4 Conclusions

A preliminary application of the FastICA algorithm to test the capability of the method in a source localization problem has been described. Under different signal to noise conditions it works properly giving results according to the simulated inputs. The method has been applied only to the reconstruction of the sources position; its effectiveness on the absolute intensity will be further investigated.

References

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